B.Sc-part-II (H), paper-III

24-07-2020.

Q.1 Find the pedal eqn of $r^m = a^m \cos m\theta$

Soln: It is given that

$$r^m = a^m \cos m\theta$$

Taking logarithm on both sides, we get

$$m\log r = m\log a + \log\cos m\theta$$

Diff. w.r. to θ, $\frac{m}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta} \times (-\sin m\theta) \times m$

$$\therefore \frac{1}{r}\frac{dr}{d\theta} = -\tan m\theta$$

$$\Rightarrow \cot\phi = -\tan m\theta = \cot\left(\frac{\pi}{2} + m\theta\right)$$

$$\therefore \phi = \frac{\pi}{2} + m\theta$$

Now, $p = r\sin\phi = r\sin\left(\frac{\pi}{2} + m\theta\right) = r\cos m\theta = r\frac{r^m}{a^m} = \frac{r^{m+1}}{a^m}$

Hence, the required pedal equation is $p = \frac{r^{m+1}}{a^m}$.

Q.2: Find the pedal equation of the curve $r = a e^{\theta\cot\alpha}$

Soln: The curve is $r = a e^{\theta\cot\alpha}$ ——— (1)

Diff. w. r. to θ, we get

$$\frac{dr}{d\theta} = a e^{\theta\cot\alpha} \cot\alpha$$

$$\Rightarrow \frac{dr}{d\theta} = r\cot\alpha = \frac{r}{\tan\alpha} \quad [\because r = a e^{\theta\cot\alpha}]$$

We know that $\tan\phi = r\frac{d\theta}{dr} = \frac{r}{\frac{dr}{d\theta}} = \frac{r\tan\alpha}{r} = \tan\alpha$

$$\therefore \phi = \alpha$$

We know that $p = r\sin\phi$

$$\therefore p = r\sin\alpha$$

This is the required pedal eqn

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Q.3. Find the pedal eqn of the curve $r = a\theta$

Soln: The eqn of the curve is $r = a\theta$ —— (1)

Diff. w.r. to θ, we get

$$\frac{dr}{d\theta} = a \Rightarrow \frac{d\theta}{dr} = \frac{1}{a}$$

But $\tan\phi = r\frac{d\theta}{dr} = \frac{r}{a}$

Also, $p = r\sin\phi = r\frac{\sin\phi}{\cos\phi\sec\phi} = \frac{r\tan\phi}{\sqrt{1+\tan^2\phi}}$

$$\Rightarrow p = \frac{\frac{r^2}{a}}{\sqrt{1+\frac{r^2}{a^2}}} = \frac{r^2}{\sqrt{r^2+a^2}}$$

$\Rightarrow p^2(a^2+r^2) = r^2$,

which is the required pedal eqn.

Q.4. Find the pedal eqn of the curve $r = a e^{m\theta}$

Soln. The eqn of the curve is $r = a e^{m\theta}$

Diff. w.r. to θ, we get

$$\frac{dr}{d\theta} = am\, e^{m\theta} = rm \quad [\because r = e^{m\theta}]$$

$\therefore \frac{1}{r}\frac{dr}{d\theta} = m \Rightarrow \cot\phi = m$

Now, $p = r\sin\phi$

$$= \frac{r}{\operatorname{cosec}\phi} = \frac{r}{\sqrt{1+\cot^2\phi}} = \frac{r}{\sqrt{1+m^2}}$$

$\therefore r^2 = (1+m^2)p^2$

which is the required pedal eqn.